March 2025

Mathematics & Statistics (Commerce)

Paper J-316 · 80 marks · 3 hours — “LPP corners, Poisson defects, a 73-day bill”

8 key questions solved step by step · 10-question answer key · 8 flashcards

Official board question paper (board portal PDF) - transcribed; solutions re-derived and machine-checked.

Section	Coverage
Section I	Q.1–Q.3 · Mathematical Logic, Matrices, Differentiation, Applications of Derivatives, Integration, Definite Integrals, Differential Equations
Section II	Q.4–Q.6 · Commission, Insurance and Annuity, Linear Programming, Sequencing, Regression, Time Series, Index Numbers, Probability Distributions

Key questions — solved in full

Q.3 (A) (iii) · 3 marks

Differential Equations · Free of constants · easy

Q. Find the differential equation by eliminating arbitrary constants from the relation $y = Ae^{3x} + Be^{-3x}$.

1. $y = Ae^{3x} + Be^{-3x}$ (A, B arbitrary constants)

↳ Two constants live in this relation, so we plan two differentiations. Watch the rhythm. Each derivative pulls down a three, and the sign of the B term flips every single time.

2. $dy/dx = 3Ae^{3x} - 3Be^{-3x}$

↳ First derivative now. The A term gives three A e to the three x. The B term carries a negative power, so its three comes down with a minus sign attached.

3. $(d^2y)/(dx^2) = 9Ae^{3x} + 9Be^{-3x}$

↳ Second derivative. Each term again takes one more factor of three. Notice the signs have flipped back, matching the original relation, and both constants now sit behind a nine.

4. $(d^2y)/(dx^2) = 9y$

↳ Compare with the starting line. Nine A e to the three x plus nine B e to the minus three x is exactly nine times y. That observation is the whole question.

5. $(d^2y)/(dx^2) - 9y = 0$

↳ Bring everything to the left side and both arbitrary constants vanish. This is the differential equation the board wants, clean and free of A and B.

ANSWER

$(d^2y)/(dx^2) - 9y = 0$

Exam rule — y = Ae^{αx} + Be^{−αx} always eliminates to y″ − α²y = 0; differentiate once per constant.

Q.3 (C) (i) · 4 marks

Applications of Derivatives · Maxima (Activity) · medium

Q. Divide the number 84 into two parts such that the product of one part and square of the other is maximum. (Complete the activity.)

1. Parts: x and 84 - x; $f(x) = x^2(84 - x) = 84x^2 - x^3$

↳ Complete the activity with me. Let one part be x, then the other part is eighty four minus x. Square of one part times the other gives our function.

2. $f'(x) = 168x - 3x^2 = 0 \Rightarrow 3x(56 - x) = 0 \Rightarrow x = 0,\ 56$

↳ Extreme values need the first derivative zero. Differentiate: one sixty eight x minus three x square. Take three x common and each factor gives a critical point.

3. $f''(x) = 168 - 6x$; $f''(0) = 168 > 0$ (min), $f''(56) = -168 < 0$ (max)

↳ Now decide maximum or minimum with the second derivative. One sixty eight minus six x. At zero it is positive, so minimum there. At fifty six it is negative, so maximum.

4. Parts: 56 and 28

↳ So the winning split is fifty six and twenty eight, since eighty four minus fifty six leaves twenty eight. Both parts of the number are settled.

5. $f_{max} = 56^2 \times 28 = 87808$

↳ Finish the activity by reporting the maximum product itself. Fifty six squared, then multiplied by twenty eight, gives eighty seven thousand eight hundred and eight exactly.

ANSWER

Parts 56 and 28; maximum product = $56^2 \times 28 = 87808$.

Exam rule — Max-min word problem: express f in ONE variable, set f′ = 0, confirm with the sign of f″.

Q.5 (A) (i) · 3 marks

Regression · Appropriate equation · easy

Q. Compute the appropriate regression equation for the following data: X: 1, 2, 3, 4, 5; Y: 5, 7, 9, 11, 13. X is the independent variable and Y is the dependent variable.

1. n = 5, Σ x = 15, Σ y = 45, Σ xy = 155, Σ x^2 = 55

↳ X is the independent variable and Y the dependent one, so the appropriate equation is the regression of Y on X. First the totals, all five of them, from the table.

2. byx = (nΣ xy - Σ x Σ y)/(nΣ x^2 - (Σ x)²) = (775 - 675)/(275 - 225) = 100/50 = 2

↳ Slope b y x by the standard formula. Numerator five times one fifty five minus fifteen times forty five. Denominator five times fifty five minus fifteen squared. Watch the cancellation.

3. $\bar{x}$ = 3, $\bar{y}$ = 9

↳ Means next. X bar is fifteen divided by five, three. Y bar is forty five divided by five, nine. Every regression line passes through this mean point, remember.

4. y - 9 = 2(x - 3) ⇒ y = 2x + 3

↳ Regression of Y on X says y minus y bar equals b y x into x minus x bar. Substitute the three numbers and simplify to a tidy line.

5. y(1) = 5, y(5) = 13 ✓ (byx · $b_{x}y$ = 2 × ½ = 1 = r^2)

↳ Test against the data before you close the answer. At x one we get five, at x five we get thirteen. Every plotted point sits on the line, so correlation is perfect.

ANSWER

y = 2x + 3 (regression of Y on X); byx = 2, r = 1.

Exam rule — Independent X, dependent Y means y-on-x only: b_yx = (nΣxy − ΣxΣy)/(nΣx^2 − (Σx)²).

Q.5 (B) (ii) · 4 marks

Sequencing · Johnson's rule, two machines · hard

Q. Find the sequence that minimizes the total elapsed time to complete the following jobs in the order AB. Find the total elapsed time and idle time for machine B. Jobs I–VII; Machine A: 7, 16, 19, 10, 14, 15, 5; Machine B: 12, 14, 14, 10, 16, 5, 7.

1. A: 7, 16, 19, 10, 14, 15, 5; B: 12, 14, 14, 10, 16, 5, 7

↳ Seven jobs pass through machine A first, then machine B, no jumping of the queue. Johnson's rule hunts the smallest time in the whole table. Machine A times go early, machine B times go late.

2. VII → 1st, VI → 7th, I → 2nd

↳ Place the extremes. Smallest five belongs to job seven on A, so job seven opens. Five on B sends job six to the end. Seven on A puts job one second.

3. Sequence: VII, I, IV, V, II, III, VI

↳ Keep going. Ten appears in both columns for job four, take it from the A side into slot three. Fourteen on A gives job five the middle seat, then three before two from the B side.

4. VII I IV V II III VI

A 0-5 5-12 12-22 22-36 36-52 52-71 71-86

B 5-12 12-24 24-34 36-52 52-66 71-85 86-91

↳ Now the passage table. Machine A never rests, zero to eighty six. Machine B starts at five, waits in three small gaps, and stops at ninety one. Read the B column carefully.

5. Total elapsed T = 91 hrs; Idle time for B = 91 - 78 = 13 hrs

↳ Totals: elapsed time ninety one hours. Machine B idles five at the start, then two, then five, then one, adding to thirteen. The question demands both figures, so write both.

ANSWER

Sequence VII, I, IV, V, II, III, VI; total elapsed time = 91 hours; idle time for B = 13 hours.

Exam rule — Johnson's rule: smallest time decides — A-column jobs to the front, B-column jobs to the back; idle B = elapsed − ΣB.

Q.6 (A) (i) · 3 marks

Insurance · Premium and commission · easy

Q. A house valued at ₹8,00,000 is insured at 75% of its value. If the rate of premium is 0.80%, find the premium paid by the owner of the house. If agent's commission is 9% of the premium, find agent's commission.

1. Policy value = 75% × 8,00,000 = 6,00,000

↳ Insurance never covers the full value of a house, only the policy amount. Seventy five percent of eight lakh rupees gives six lakh rupees of cover. That is our base for the premium.

2. Premium = 6,00,000 × (0.80)/(100) = 4,800

↳ Premium runs at zero point eight percent of the policy value. Six lakh into zero point eight by one hundred gives four thousand eight hundred rupees. Simple unitary method, nothing fancy.

3. Agent's commission = 9% of 4800

↳ The agent earns nine percent of the premium, not of the house value. Students mix these two bases every year in the board exam, so mark this line.

4. Agent's commission = 9/100 × 4800 = 432

↳ Nine percent of four thousand eight hundred comes to four hundred thirty two rupees. Two answers down, premium and commission, three clean marks in the pocket.

ANSWER

Policy value ₹6,00,000; premium ₹4,800; agent's commission ₹432.

Exam rule — Premium is charged on the POLICY value (value insured), never on the full property value; agent's cut is on the premium.

Q.6 (A) (ii) · 4 marks

Linear Programming · Graphical method · medium

Q. Solve the following L.P.P. by graphical method: Maximize z = 4x + 6y subject to 3x + 2y ≤ 12, x + y ≥ 4, x ≥ 0, y ≥ 0.

1. 3x + 2y = 12: (4,0),\ (0,6); x + y = 4: (4,0),\ (0,4)

↳ Draw both boundary lines as equalities first. Three x plus two y equals twelve meets the axes at four zero and zero six. x plus y equals four meets them at four zero and zero four.

2. Corners: (0,4),\ (0,6),\ (4,0)

↳ Shade below the first line and above the second, first quadrant only. The two lines meet at four zero itself, so the feasible region is one tidy triangle with three corners.

3. z(0,4) = 24, z(0,6) = 36, z(4,0) = 16

↳ Corner point theorem time, our Main Event rule. Test each corner of the region in z equals four x plus six y. Only three corners, so three quick multiplications and two additions each.

4. zmax = 36 at (0, 6)

↳ The largest value is thirty six, at zero six. The optimum of a linear programme always sits at a corner of the feasible region, never inside it.

ANSWER

zmax = 36 at x = 0, y = 6.

Exam rule — Intersect the boundaries for corners, evaluate z at each corner, take the max — the optimum lives at a vertex.

Q.6 (A) (iii) · 3 marks

Probability Distributions · Poisson · medium

Q. Defects on plywood sheets occur at random with the average of one defect per 50 sq.ft. Find the probability that such a sheet has: (a) no defect (b) at least one defect (use e^{-1} = 0.3678).

1. P(X = r) = (e^{-m} m^(r))/(r!), m = 1

↳ Defects occurring at random with a fixed average point to the Poisson distribution. One defect per fifty square feet, and our sheet is fifty square feet, so the mean m is one.

2. P(X = 0) = e^{-1} = 0.3678

↳ No defect means r equals zero. m to the zero is one, factorial zero is one, so the whole formula collapses to e to the minus one. Use the printed value only.

3. P(X ≥ 1) = 1 - P(X = 0)

↳ Part b is the complement trick. At least one defect plus no defect covers everything, so their probabilities add to one. Subtract from one, no integration, no table.

4. P(X ≥ 1) = 1 - 0.3678 = 0.6322

↳ One minus zero point three six seven eight gives zero point six three two two. Both parts done in three lines each, and the e value stays exactly as printed.

ANSWER

(a) P(no defect) = 0.3678; (b) P(at least one defect) = 0.6322.

Exam rule — Random defects with mean m per unit → Poisson; P(X ≥ 1) = 1 − e^{−m}.

Q.6 (B) (ii) · 4 marks

Index Numbers · Cost of living index · medium

Q. Find x if the cost of living index is 150: Group: Food, Clothing, Fuel and electricity, House Rent, Miscellaneous; I: 180, 120, 300, 100, 160; W: 4, 5, 6, x, 3.

1. CLI = (Σ IW)/(Σ W)

↳ Cost of living index is a weighted average of the group index numbers. Sum of I into W divided by sum of W. The unknown x sits in the weight column, so keep it as a letter.

2. Σ IW = 720 + 600 + 1800 + 100x + 480 = 3600 + 100x

↳ Multiply each group index by its weight and add. Seven twenty, six hundred, eighteen hundred, one hundred x, and four eighty. The numerator collects into thirty six hundred plus one hundred x.

3. (3600 + 100x)/(18 + x) = 150

↳ Weights total four plus five plus six plus x plus three, which is eighteen plus x. Now set the whole fraction equal to one fifty, the given index.

4. 3600 + 100x = 2700 + 150x ⇒ 50x = 900 ⇒ x = 18

↳ Cross multiply. Thirty six hundred plus one hundred x equals twenty seven hundred plus one fifty x. Fifty x equals nine hundred, so x is eighteen.

5. Check: 5400/36 = 150 ✓

↳ Always verify before closing: numerator fifty four hundred, denominator thirty six, ratio exactly one fifty. The house rent group carries weight eighteen, and the answer is sealed.

ANSWER

x = 18

Exam rule — CLI = ΣIW ÷ ΣW. Put the unknown weight in, cross multiply, then verify the given index returns.

Objective answer key — March 2025

1. [PYQ · March 2025] An agent who gives a guarantee to his principal that the party will pay the sale price of goods is called _____.

(A) Auctioneer **(B)** Del credere agent (C) Factor (D) Broker

A del credere agent guarantees collection of the sale price from the buyer, in return for extra commission. Asked verbatim in March 2025 Q.4 (A) (i).

2. [PYQ · March 2025] In an ordinary annuity, payments or receipts occur at the _____.

(A) Beginning of each period **(B)** End of each period (C) Mid of each period (D) Quarterly basis only

Ordinary annuity means payments at the END of each period; an annuity due has them at the beginning. March 2025 objective question.

3. [PYQ · March 2025] Moving averages are useful in identifying the _____ component of a time series.

(A) Seasonal (B) Irregular **(C)** Trend (D) Cyclical

Averaging smooths short-run ups and downs and brings out the underlying trend (secular variation). Direct PYQ from Q.4 (A) (iii).

4. [PYQ · March 2025] The objective of an assignment problem is to assign _____.

(A) Number of jobs to equal number of persons at maximum cost **(B)** Number of jobs to equal number of persons at minimum cost (C) Only to maximize the number of persons (D) Only to minimize the number of jobs

One-to-one assignment of n jobs to n persons so that total cost (or time) is minimum. March 2025 Q.4 (A) (v).

5. [PYQ · March 2025] The expected value of the sum of the numbers obtained when two fair dice are rolled is _____.

(A) 5 (B) 6 **(C)** 7 (D) 8

Each die has expectation 3.5, so by linearity E(sum) = 3.5 + 3.5 = 7. Verified by enumerating all 36 outcomes.

6. [PYQ · March 2025] If $b_{yx} \times b_{xy} = 1.30$ and r = 0.75, the data is inconsistent. This statement is _____.

(A) True (B) False (C) True only if r is negative (D) Cannot be decided

Consistency demands $b_{yx} \cdot b_{xy} = r^2$. Here $r^2 = 0.5625$ but the product is 1.30, so the data is inconsistent — the statement is true.

7. [PYQ · March 2025] The amount paid to the holder of a bill after deducting banker's discount is known as _____.

(A) Face value (B) Banker's gain **(C)** Cash value (D) Legal due date

The banker deducts BD from the sum due (face value); the balance actually paid in cash is the cash value (present worth). Fill-in from Q.4 (C) (i).

8. [PYQ · March 2025] The area of the region bounded by the curve $y = x^2$ and the line y = 4 is _____ sq units.

(A) 32/3 (B) 64/3 (C) 16/3 (D) 64

Curves meet at $x = \pm 2$; area = $\int$ from -2 to 2 of $(4 - x^2)\,dx = 32/3$ sq units. March 2025 Q.1 (A) (iv).

9. [Practice · Commission, Insurance and Annuity] The legal due date of a bill of exchange is obtained by adding _____ days of grace to the nominal due date.

(A) 2 **(B)** 3 (C) 5 (D) 7

Three days of grace are added to the nominal due date; the March 2025 bill activity used this to count exactly 73 discount days.

10. [Practice · Applications of Derivatives] Average revenue is 50 and elasticity of demand is 5. Marginal revenue is _____.

(A) 40 (B) 45 (C) 48 (D) 50

MR = $AR(1 - 1/\eta) = 50 \times (1 - 1/5) = 40$. Reworded from the March 2025 fill-in on revenue and elasticity.

Flashcards — March 2025

Front	Recall line
Del credere agent	Guarantees the principal that buyers will pay the sale price
Ordinary annuity	Payments at the END of each period (annuity due: beginning)
Poisson P(X = 0)	e^{-m}; so P(X ≥ 1) = 1 − e^{-m}
b_yx × b_xy	Must equal r²; a mismatch means inconsistent data
Cost of living index	CLI = ΣIW ÷ ΣW; used for purchasing power of money
Johnson's rule	Smallest time first: A-column jobs early, B-column jobs late
Expected sum of two dice	3.5 + 3.5 = 7 (linearity of expectation)
y = Ae^{3x} + Be^{-3x}	Eliminates to y″ − 9y = 0