March 2024

# Mathematics & Statistics (Commerce)

Paper J-866 · 80 marks · 3 hours — “Which corner hides the 2024 minimum?”

8 key questions solved step by step · 10-question answer key · 8 flashcards

Official board question paper (board portal PDF) - transcribed; solutions re-derived and machine-checked.

| Section | Coverage |
|---|---|
| **Section I** | Q.1–Q.3 · Mathematical Logic, Matrices, Differentiation, Integration, Differential Equations |
| **Section II** | Q.4–Q.6 · Commission, Insurance & Annuity, Regression, Index Numbers, LPP, Sequencing, Probability Distributions |

# Key questions — solved in full

## Q.2 (B) (ii) · 4 marks

Applications of Derivatives · Minimum value · medium

**Q. Determine the minimum value of the function : $f(x) = 2x^3 - 21x^2 + 36x - 20$.**

**1.** $f'(x) = 6x^2 - 42x + 36 = 6(x^2 - 7x + 6) = 6(x-1)(x-6)$

↳ Minimum value means we interrogate the derivative first. Differentiate the cubic term by term, pull out the common six, then factorise the quadratic inside. Watch how neatly it splits.

**2.** $f'(x) = 0 \Rightarrow x = 1$ or $x = 6$

↳ Stationary points arrive when the derivative equals zero. So x is one or x is six. Two candidates only, and the smaller one will soon prove greedy, giving the maximum.

**3.** $f''(x) = 12x - 42$; $f''(1) = -30 < 0$, $f''(6) = 30 > 0 \Rightarrow$ minimum at $x = 6$

↳ Second derivative test decides the winner. f double prime is twelve x minus forty two. At one it is minus thirty, negative, so maximum there. At six it is plus thirty, positive, and that is our minimum.

**4.** $f(6) = 2(216) - 21(36) + 36(6) - 20 = 432 - 756 + 216 - 20$

↳ Now feed x equals six into the original function. Two times two sixteen is four hundred thirty two. Twenty one times thirty six is seven hundred fifty six. Then plus two sixteen and minus twenty.

**5.** $f(6) = -128$

↳ Add gently. Four hundred thirty two plus two hundred sixteen makes six hundred forty eight. Six forty eight minus seven fifty six, then minus twenty, lands on minus one hundred twenty eight. That is the minimum value.

ANSWER

**Minimum value = f(6) = -128, attained at x = 6.**

Exam rule — Second derivative test: f double prime positive at the critical point gives the minimum value.

## Q.3 (A) (ii) · 3 marks

Integrals · Recovering f(x) · easy

**Q. If $f'(x) = 4x^3 - 3x^2 + 2x + k$, $f(0) = 1$ and $f(1) = 4$, find $f(x)$.**

**1.** $f(x) = \int (4x^3 - 3x^2 + 2x + k)\,dx = x^4 - x^3 + x^2 + kx + c$

↳ We climb back from the derivative to the function. Integrate term by term: four x cubed returns x to the fourth, three x squared returns x cubed, and so on. Do not forget the constant.

**2.** $f(0) = c = 1$

↳ First condition, f of zero is one. Put x equal to zero and every power dies except the constant c. So c is one, the first unknown falls.

**3.** $f(1) = 1 - 1 + 1 + k + 1 = 2 + k = 4$

↳ Second condition, f of one is four. Substitute: one minus one plus one plus k plus one. Three ones cancel to two, leaving two plus k equals four.

**4.** $k = 2 \Rightarrow f(x) = x^4 - x^3 + x^2 + 2x + 1$

↳ So k is two. Assemble the complete function with every coefficient in place, then silently re-differentiate in your head to confirm nothing was lost on the way.

ANSWER

**$f(x) = x^4 - x^3 + x^2 + 2x + 1$**

Exam rule — Antiderivative plus two conditions: the first fixes c, the second fixes k.

## Q.3 (B) (i) · 4 marks

Matrices · Adjoint method · medium

**Q. Find the inverse of bmatrix 3 & 1 & 5 2 & 7 & 8 1 & 2 & 5 bmatrix by adjoint method.**

**1.** $|A| = 3(35 - 16) - 1(10 - 8) + 5(4 - 7) = 57 - 2 - 15 = 40 \neq 0$

↳ Adjoint method recipe: determinant first, then all nine cofactors, transpose them, divide by the determinant. Expand along row one. Three times nineteen, minus one times two, plus five times minus three.

**2.** $C_{11}=19$, $C_{12}=-2$, $C_{13}=-3$, $C_{21}=5$, $C_{22}=10$, $C_{23}=-5$, $C_{31}=-27$, $C_{32}=-14$, $C_{33}=19$

↳ Cofactors now, all nine, each a two by two minor with an alternating sign. Move slowly here, one careless sign ruins the whole inverse. Read them row by row.

**3.** adj A = bmatrix 19 & 5 & -27
-2 & 10 & -14
-3 & -5 & 19 bmatrix
↳ The adjoint is the transpose of the cofactor matrix. Rows become columns, so nineteen, five and minus twenty seven stack in the first column. Keep the brackets tidy.

**4.** $A^{-1}$ = 1/40bmatrix 19 & 5 & -27
-2 & 10 & -14
-3 & -5 & 19 bmatrix, A $A^{-1}$ = I ✓
↳ Inverse equals adjoint divided by determinant, so every entry sits over forty. Finish like a topper: multiply A with its inverse and confirm the identity matrix appears. It does.

ANSWER

**$A^{-1}$ = 1/40bmatrix 19 & 5 & -27**
**-2 & 10 & -14**
**-3 & -5 & 19 bmatrix**

Exam rule — Adjoint method: determinant, cofactor matrix, transpose, then divide by the determinant.

## Q.5 (A) (i) · 3 marks

Commission · Salary plus commission · easy

**Q. Deepak's salary was increased from ₹4,000 to ₹5,000. The sales being the same, due to reduction in the rate of commission from 3% to 2%, his income remains unchanged. Find his sales.**

**1.** Old income = 4000 + 3/100s, New income = 5000 + 2/100s
↳ Let the sales be s rupees. Old income: four thousand salary plus three percent commission. New income: five thousand salary plus two percent commission. Read the question twice: same sales, equal incomes.

**2.** 4000 + 3/100 s = 5000 + 2/100 s
↳ The incomes are equal, so equate the two expressions. That single equation carries the whole question. Write it cleanly before moving any term. No second equation is needed here.

**3.** (3s - 2s)/(100) = 1000 ⇒ s/100 = 1000
↳ Bring the commission terms to one side and salaries to the other. Three percent minus two percent of s equals one thousand rupees. That is simply one percent of s.

**4.** s = 1,00,000; 4000 + 3000 = 7000 = 5000 + 2000 ✓
↳ One percent of s is one thousand, so s is one lakh rupees. Check it: old income seven thousand, new income seven thousand. Equal, confirmed, full marks.

ANSWER

**Sales = ₹1,00,000.**

Exam rule — Unchanged income: build both salary-plus-commission expressions and equate them.

## Q.5 (A) (ii) · 3 marks

Linear Regression · Variance from the b's · medium

**Q. For a bivariate data, the regression co-efficient of Y on X is 0.4 and the regression co-efficient of X on Y is 0.9. Find the value of variance of Y if variance of X is 9.**

**1.** $r^2$ = b_(YX) · b_(XY) = 0.4 × 0.9 = 0.36
↳ Two regression coefficients and one variance, a three step duet. The golden link first: the product of the two b's is r squared. Multiply zero point four by zero point nine.

**2.** r = +0.6 (both b's positive)
↳ Square root of zero point three six gives zero point six. Both b's are positive, so r keeps the positive sign. Remember, r never disagrees with its two parents.

**3.** b_(YX) = r ()/($\sigma_x$) ⇒ 0.4 = 0.6 × ()/(3) ⇒ = 2
↳ Now the definition: b y x equals r times sigma y over sigma x. Sigma x is the square root of nine, three. Slide the numbers and sigma y comes out as two.

**4.** Var(Y) = ² = 4

↳ Variance is sigma squared, so variance of y is four. One line of caution for the exam: variance of x was nine, do not plug nine where three belongs.

ANSWER

**Var(Y) = 4 (with r = 0.6)**

Exam rule — b_YX times b_XY gives r squared; the sign of r matches the common sign of both b's.

## Q.5 (B) (ii) · 4 marks

Sequencing · Three machines in order ABC · hard

**Q. A toy manufacturing company produces five types of toys. Each toy has to go through three machines A, B and C in the order ABC. The time required in hours: Type 1, 2, 3, 4, 5 need Machine A 16, 20, 12, 14, 22; Machine B 10, 12, 4, 6, 8; Machine C 8, 18, 16, 12, 10. Find the total elapsed time and also find idle time for machine B.**

**1.** min A + min B = 12 + 4 = 16 ≥ max B = 12; min C + min B = 8 + 4 = 12 ≥ 12 ✓

↳ Three machines A, B, C in that order. Before merging, clear the pass test: minimum of A plus minimum of B must reach maximum of B, and same for C. Sixteen passes, twelve passes, both clear.

**2.** G = A + B: 26, 32, 16, 20, 30 and H = B + C: 18, 30, 20, 18, 18

↳ Condition satisfied, so collapse to two pretend machines. G is A plus B for every toy, H is B plus C. Write both rows neatly before touching the sequence.

**3.** Optimal sequence: 3 → 2 → 5 → 4 → 1

↳ Two machine rule now. Smallest time sits at its end: sixteen belongs to toy three on G, so toy three goes first. The eighteen times on H pull toys five, four and one to the back. Toy two fills the middle.

**4.** B runs 12–16, 32–44, 54–62, 68–74, 84–94; C finishes at 102 hours

↳ Now roll the real machines with order three, two, five, four, one. A never waits, finishing at eighty four. B starts at twelve and keeps pausing. C trails, and the last toy leaves C at one hundred two.

**5.** Total elapsed time = 102 hours; Idle time for B = 102 - 40 = 62 hours

↳ Total elapsed time is one hundred two hours. Machine B works only forty hours in all, so its idle time is one hundred two minus forty, sixty two hours. Quote both numbers with units.

ANSWER

**Sequence 3-2-5-4-1; total elapsed time = 102 hours; idle time for machine B = 62 hours.**

Exam rule — Three machines: verify min plus min against max B, merge to G and H, sequence, then simulate the real times.

## Q.5 (B) (iii) · 4 marks

Probability Distributions · Finding k · medium

**Q. A random variable X has the following probability distribution : x: 1 to 7 with P(x) = k,\ 2k,\ 2k,\ 3k,\ k²,\ 2k²,\ 7k² + k. Find (a) k (b) P(X < 3) (c) P(X > 6).**

**1.** k + 2k + 2k + 3k + k² + 2k² + 7k² + k = 1

↳ One law opens every distribution question: total probability is one. Add the four k terms and the three k squared terms, and do not drop the little plus k hiding at the end.

**2.** 10k² + 9k = 1 ⇒ 10k² + 9k - 1 = 0

↳ Count the k's: one plus two plus two plus three plus one more gives nine k. The squares: one plus two plus seven gives ten k squared. A tidy quadratic in k.

**3.** (10k - 1)(k + 1) = 0 ⇒ k = 1/10 (k = -1 rejected)

↳ Factorise: ten k minus one times k plus one. k is one tenth or minus one. Probabilities can never be negative, so minus one walks out of the room.

**4.** P(X < 3) = k + 2k = 3k = 3/10

↳ P of X less than three covers x one and x two only. k plus two k is three k, that is three tenths. The strict sign keeps x three outside the gate.

**5.** P(X > 6) = 7k² + k = 7/100 + 10/100 = 17/100

↳ P of X greater than six means only x seven. Its entry is seven k squared plus k, which is seven hundredths plus ten hundredths. Seventeen hundredths, and the question is done.

ANSWER

**k = 0.1; P(X < 3) = 0.3; P(X > 6) = 0.17**

Exam rule — Total probability equal to one fixes k; reject negative roots, then read strict inequalities entry by entry.

## Q.6 (B) (ii) · 4 marks

Linear Programming · Graphical method (minimize) · medium

**Q. Minimize : z = 6x + 2y, Subject to : x + 2y ≥ 3, x + 4y ≥ 4, 3x + y ≥ 3, x ≥ 0, y ≥ 0.**

**1.** x + 2y = 3, x + 4y = 4, 3x + y = 3; feasible region on the far side of each line

↳ All three constraints push us away from the origin, greater than or equal, so the feasible region runs unbounded and we are minimizing. Draw the three boundary lines in the first quadrant only.

**2.** x+2y = 3 meets x+4y = 4 at (2, ½); x+2y = 3 meets 3x+y = 3 at (⅗, 6/5)

↳ Corner points come from pairwise meetings. The y axis meets three x plus y equal three at zero three. The x axis meets x plus four y equal four at four zero. Inside, two more crossings.

**3.** z(0,3)=6, z(⅗,6/5)=6, z(2,½)=13, z(4,0)=24

↳ Four corners, that is all. Test each in z equals six x plus two y. At zero three we get six. At three fifths six fifths, again six. At two and a half, thirteen. At four zero, twenty four.

**4.** z = 2(3x + y) ∥ 3x + y = 3 ⇒ multiple optima on the whole edge

↳ Look, two corners tie at six. The reason is lovely: z equals two times three x plus y, exactly double the third boundary line. So every point on that edge gives six, infinitely many optima.

**5.** z_(min) = 6 at every point of the segment joining (0, 3) and (⅗, 6/5)

↳ Minimum value of z is six, attained all along the segment joining zero three and three fifths six fifths. In the board answer, state both corners and say the optima are many.

ANSWER

**z_(min) = 6, at every point of the segment joining (0, 3) and (⅗, 6/5) (multiple optima).**

Exam rule — Objective parallel to a boundary line means multiple optimal solutions; state the entire optimal segment.

# Objective answer key — March 2024

**1. [PYQ · March 2024]** Which of the following is NOT a statement in logic?

(A) Smoking is injurious to health (B) 2 + 2 = 4 (C) 2 is the only even prime number **(D)** Come here

A statement must be either definitely true or definitely false. 'Come here' is a command, so it is not a statement.

**2. [PYQ · March 2024]** If x + y + z = 3, x + 2y + 3z = 4 and x + 4y + 9z = 6, then (y, z) = ?

(A) (-1, 0) **(B)** (1, 0) (C) (1, -1) (D) (-1, 1)

Subtracting successive equations gives y + 2z = 1 and y + 3z = 1, so z = 0, y = 1 (then x = 2 satisfies all three).

**3. [PYQ · March 2024]** ∫ (dx)/(√(1 - x)) = ?

(A) 2√(1 - x) + c **(B)** -2√(1 - x) + c (C) √(x) + c (D) x + c

Write the integrand as (1-x)^(-1/2); integrating gives ((1-x)^(1/2))/(1/2) · (-1) = -2√(1-x). Differentiate back to confirm.

**4. [PYQ · March 2024]** The differential equation of $y = k_1 e^x + k_2 e^{-x}$ is :

**(A)** $(d^2y)/(dx^2) - y = 0$ (B) $(d^2y)/(dx^2) + dy/dx = 0$ (C) $(d^2y)/(dx^2) + ydy/dx = 0$ (D) $(d^2y)/(dx^2) + y = 0$

Two arbitrary constants mean a second order equation. Differentiating twice gives $y'' = k_1e^x + k_2e^{-x} = y$, so $y'' - y = 0$.

**5. [PYQ · March 2024]** The difference between the face value and the present worth of a bill is called _____.

(A) Banker's discount **(B)** True discount (C) Banker's gain (D) Cash value

True discount is the present worth of the face value; equivalently, face value minus present worth equals the true discount.

**6. [PYQ · March 2024]** In an ordinary annuity, payments or receipts occur at _____.

(A) beginning of each period **(B)** end of each period (C) mid of each period (D) quarterly basis

Ordinary annuity pays at the END of each period; the beginning-of-period version is an annuity due.

**7. [Practice · Linear Regression]** $b_{xy}$ and byx are _____.

(A) independent of change of origin and scale **(B)** independent of change of origin but not of scale (C) independent of change of scale but not of origin (D) affected by change of origin and scale

Regression coefficients are unchanged by a shift of origin but change when the scale is altered.

**8. [Practice · Linear Programming]** The optimum value of the objective function of an LPP occurs at the _____ of the feasible region.

(A) center **(B)** corner (vertex) (C) midpoint of the longest edge (D) origin, always

Corner Point Theorem: the optimal value of z is attained at a vertex of the feasible region (the paper tested this as true or false).

**9. [Practice · Integrals]** If f'(x) = 1/x + x and f(1) = 5/2, then f(x) = log x + $(x^2)/(2)$ + _____.

(A) 1 **(B)** 2 (C) ½ (D) 0

Substitute x = 1: f(1) = 0 + ½ + c = 5/2 gives c = 2. The given value pins the integration constant.

**10. [Practice · Binomial Distribution]** Eggs are drawn with replacement from a lot containing 10 percent defective eggs. The probability of at least one defective egg among 10 eggs is _____.

(A) $(0.9)^{10}$ **(B)** 1 - $(0.9)^{10}$ (C) $(0.1)^{10}$ (D) 1 - $(0.1)^{10}$

Binomial with n = 10, p = 0.1: P(none defective) = $(0.9)^{10}$, so at least one defective = 1 - $(0.9)^{10}$ ≈ 0.651.

## Flashcards — March 2024

| Front | Recall line |
|---|---|
| **Adjoint method** | Inverse = adj A over det A; check A times inverse gives I |
| **b_YX times b_XY** | Gives r squared; r takes the common sign of both b's |
| **Three-machine sequencing** | Pass test, merge to G = A+B and H = B+C, sequence, simulate real times |
| **Finding k in a pmf** | Set total probability equal to one; reject negative roots |
| **LPP multiple optima** | Objective parallel to a boundary edge gives a whole optimal segment |
| **Second derivative test** | f double prime negative means maximum, positive means minimum |
| **Commission income** | Salary plus rate times sales; equate old and new incomes |
| **Banker's discount** | Always greater than the true discount |