March 2023

Mathematics & Statistics (Commerce)

Paper Comm-88 · 80 marks · 3 hours — “Where the 2023 paper taught patience”

8 key questions solved step by step · 10-question answer key · 8 flashcards

Verified transcription of the official March 2023 board paper (subject 88). Solutions re-derived and machine-checked.

Section	Coverage
Section I	Q.1–Q.3 · Logic, Matrices, Differentiation, Integration, Differential Equations
Section II	Q.4–Q.6 · Commission, Regression, Index Numbers, LPP, Assignment, Probability

Key questions — solved in full

Q.2 (B) (i) · 4 marks

Matrices · Reduction method · medium

Q. Solve the following equations by reduction method: x + 2y + z = 8, 2x + 3y - z = 11, 3x - y - 2z = 5.

1. Augmented matrix [
1 2 1 8
2 3 -1 11
3 -1 -2 5]

↳ Students, watch how one matrix eats three equations. We write the augmented matrix, then sweep rows until the bottom row gives us z directly.

2. $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1 \Rightarrow$ [
1 2 1 8
0 -1 -3 -5
0 -7 -5 -19]

↳ First sweep. Subtract two times row one from row two, and three times row one from row three. The first column goes clean.

3. $R_3 \to R_3 - 7R_2 \Rightarrow$ [0 0 16 | 16] so $16z = 16$

↳ Second sweep. Row three minus seven times row two. See the x and y both vanish. One equation, one unknown.

4. $z = 1$, $-y - 3(1) = -5 \Rightarrow y = 2$

↳ So z is one. Now climb back up. From row two, minus y minus three z equals minus five. Put z equal to one, y becomes two.

5. $x + 2(2) + 1 = 8 \Rightarrow x = 3$; check $2(3)+3(2)-1 = 11$ ✓

↳ Last climb. Row one says x plus two y plus z equals eight. Substitute both values and x is three. Always verify in the original equations.

ANSWER

(x, y, z) = (3, 2, 1)

Exam rule — Reduction method = row-reduce the augmented matrix, then back-substitute from the bottom row.

Q.2 (B) (ii) · 4 marks

Applications of Derivatives · Cost & Revenue · hard

Q. For manufacturing x units, labour cost is ₹(150 - 54x) and processing cost is ₹x^2. Price of each unit is P = 10800 - 4x^2. Find the values of x for which (a) total cost is decreasing, (b) revenue is increasing.

1. $C(x) = (150 - 54x) + x^2 = x^2 - 54x + 150$

↳ Two questions, one habit: write the total function first, then differentiate. Total cost is labour plus processing.

2. $R(x) = x \cdot P = x(10800 - 4x^2) = 10800x - 4x^3$

↳ Revenue is price times units. Not the price itself, mind you. Multiply P by x before touching the derivative.

3. $C'(x) = 2x - 54 < 0 \Rightarrow x < 27$

↳ Cost decreasing means its derivative is negative. Differentiate C and solve the inequality.

4. $R'(x) = 10800 - 12x^2 > 0 \Rightarrow x^2 < 900 \Rightarrow 0 \le x < 30$

↳ Revenue increasing means its derivative is positive. Differentiate R and keep x non negative, units cannot be negative.

5. Cost decreasing: $x < 27$; Revenue increasing: $0 \le x < 30$

↳ So the factory works sweetly below twenty seven units for cost, and below thirty for revenue. Two inequalities, four marks.

ANSWER

Cost decreasing for $x < 27$; Revenue increasing for $0 \le x < 30$.

Exam rule — Decreasing or increasing always means: differentiate the TOTAL function, solve the inequality.

Q.3 (A) (ii) · 3 marks

Maxima & Minima · Geometry word problem · easy

Q. A rod of 108 meters long is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum.

1. $2(x + y) = 108 \Rightarrow y = 54 - x$

↳ The rod becomes the perimeter. One hundred eight equals two lengths plus two breadths. That is our only constraint.

2. $A = x(54 - x) = 54x - x^2$

↳ Area is length times breadth. Substitute the constraint so one variable remains. One variable, one derivative.

3. $A'(x) = 54 - 2x = 0 \Rightarrow x = 27$; $A''(x) = -2 < 0 \Rightarrow$ maximum

↳ Set the derivative to zero for the stationary point. The second derivative is minus two, negative, so this point is a maximum.

4. $x = y = 27$ m, $A_{max} = 27 \times 27 = 729$\ m^2

↳ Breadth also becomes fifty four minus twenty seven, twenty seven. Equal sides. The rectangle that wins is the square.

ANSWER

Square of side 27 m; maximum area 729 m².

Exam rule — Fixed perimeter: the square always gives maximum rectangular area.

Q.3 (B) (i) · 4 marks

Definite Integration · Area · medium

Q. Find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$.

1. $y^2 = 4x \Rightarrow y = \pm 2\sqrt{(x)}$, region $0 \le x \le 3$

↳ Draw first, integrate second. The parabola opens to the right. The line x equals three caps it. The region is symmetric above and below the x axis.

2. $A = 2 \int_0^3 2\sqrt{(x)}\, dx = 4\int_0^3 x\hat{}(1/2)\, dx$

↳ Total area is twice the upper branch. Twice the integral of two root x from zero to three.

3. $A = 4\ [⅔x\hat{}(3/2)\ ]_0^3 = 8/3\ (3\sqrt{(3)}) = 8\sqrt{(3)}$

↳ Power rule for integration. Root x integrates to two thirds x to the three by two. Evaluate at three, subtract at zero.

4. $A = 8\sqrt{(3)} \approx 13.86$\ sq units

↳ Eight root three square units, about thirteen point nine. The zero lower limit contributed nothing, as it always does here.

ANSWER

A = 8√(3) sq units ≈ 13.86

Exam rule — Symmetric region about the x-axis? Integrate the top branch and double.

Q.3 (C) (ii) · 4 marks

Differential Equations · Growth (Activity) · medium

Q. In a certain culture of bacteria, the rate of increase is proportional to the number present. If the number doubles in 4 hours, find the number of times the bacteria increase in 12 hours. (Complete the activity.)

1. $dN/dt = KN \Rightarrow dN/N = K\, dt$

↳ This was the activity question. The board gave the skeleton, you fill the blanks. Rate proportional to number present, straight to the differential equation.

2. $\log N = Kt + c$, $t=0$, $N = N_0 \Rightarrow c = \log N_0$

↳ Integrate both sides. Log N equals K t plus c. First condition, at time zero N equals N naught, so c is log N naught.

3. $\log 2 = 4K \Rightarrow K = ¼\log 2$

↳ Second condition. Doubles in four hours means N equals two N naught at t equals four. Log two equals four K. That fixes K, the blank the board wanted.

4. $\log(N)/(N_0) = (\log 2)/(4) \times 12 = 3\log 2$

↳ Now the target, twelve hours. Log N minus log N naught is K times twelve, which is three log two. Three doublings.

5. $(N)/(N_0) = 2^3 = 8 \Rightarrow N = 8N_0$

↳ Antilog both sides. N by N naught is two cubed, eight. In twelve hours the culture is eight times itself.

ANSWER

Bacteria increase 8 times in 12 hours.

Exam rule — Growth DE: dN/dt = kN. Doubling time fixes k; then scale time by doublings.

Q.5 (B) (i) · 4 marks

Linear Programming · Graphical · medium

Q. Solve the LPP by graphical method: Maximize z = 4x + 6y subject to 3x + 2y ≤ 12, x + y ≥ 4, x ≥ 0, y ≥ 0.

1. 3x + 2y = 12 (below), x + y = 4 (above), first quadrant only

↳ Two boundary lines rule this problem. Draw them as equalities, then shade the correct side of each.

2. 3x + 2(4 - x) = 12 ⇒ x = 4, y = 0

↳ Where do the lines meet? Solve the pair. Substitute y equals four minus x into the first line, x is four, y is zero.

3. Corners: (0,4),\ (0,6),\ (4,0)

↳ The feasible region is the sliver between the two lines in the first quadrant. Three corners only. Test each corner in the objective.

4. z(0,4)=24, z(0,6)=36, z(4,0)=16

↳ Corner tour. At zero four, z is twenty four. At zero six, z is thirty six. At four zero, z is sixteen.

5. zmax = 36 at (0, 6)

↳ Maximum of z is thirty six at the corner zero six. Remember the rule, the optimum lives at a corner, never inside.

ANSWER

zmax = 36 at x = 0, y = 6.

Exam rule — Corner Point Theorem: evaluate z at every corner of the feasible region.

Q.5 (B) (iii) · 4 marks

Probability Distributions · Finding k · medium

Q. A random variable X has the probability distribution: x: 1 to 7 with P(x) = k, 2k, 2k, 3k, k², 2k², 7k² + k. Find (a) k (b) P(X < 3) (c) P(X > 4).

1. ∑ P(x) = (k + 2k + 2k + 3k) + (k² + 2k² + 7k²) + k = 1

↳ One law runs this question. Total probability is one. Add all seven expressions, set equal to one, and solve.

2. 10k² + 9k = 1 ⇒ 10k² + 9k - 1 = 0

↳ Collect terms carefully. Eight k of the plain kind, plus another k, that is nine k. Ten k squared. A neat quadratic.

3. (10k - 1)(k + 1) = 0 ⇒ k = 1/10 (k = -1 rejected)

↳ Factorise. Ten k minus one times k plus one. k is one tenth or minus one. Probability cannot be negative, so reject minus one.

4. P(X < 3) = k + 2k = 3k = 3/10

↳ P of X less than three means x equals one and two. k plus two k, three k, three tenths.

5. P(X > 4) = 10k² + k = 10/100 + 1/10 = ⅕

↳ P of X greater than four is the last three entries. k squared plus two k squared plus seven k squared plus k. Ten k squared plus k. One tenth plus one tenth, one fifth.

ANSWER

k = 0.1, P(X<3) = 0.3, P(X>4) = 0.2.

Exam rule — ΣP(x) = 1 gives k. Reject negative roots; read strict inequalities entry-wise.

Q.6 (B) (i) · 4 marks

Regression · Estimation · medium

Q. From the following data, find the regression equation of y on x and estimate y when x = 4: (x, y): (1,2),\ (2,1),\ (3,6).

1. n = 3, Σ x = 6, Σ y = 9, Σ xy = 22, Σ x² = 14

↳ Regression of y on x needs five totals. n, sum x, sum y, sum xy and sum x squared. Build the table, no shortcuts.

2. $byx = (n\Sigma xy - \Sigma x \Sigma y)/(n\Sigma x^2 - (\Sigma x)^2) = (66 - 54)/(42 - 36) = 2$

↳ b yx by the formula. Three times twenty two minus six times nine over three times fourteen minus thirty six. Twelve over six, two.

3. $\bar{x} = 2, \bar{y} = 3$

↳ Means next. x bar is two, y bar is three. The regression line always passes through the mean point.

4. $y - 3 = 2(x - 2) \Rightarrow y = 2x - 1$

↳ Line of y on x. y minus y bar equals b yx times x minus x bar. Substitute and simplify to y equals two x minus one.

5. $y(4) = 2(4) - 1 = 7$

↳ Estimate time. Put x equals four. y equals seven. That is the publisher's answer for four on the x scale.

ANSWER

y = 2x - 1; estimate y(4) = 7.

Exam rule — y on x: slope b_yx, line passes through ($\bar{x}$, $\bar{y}$). Estimate by substitution only.

Objective answer key — March 2023

1. [PYQ · March 2023] If y = xlog x, then dy/dx = ?

(A) 1 (B) 1/x (C) log x **(D)** 1 + log x

Product rule: (x)′·log x + x·(1/x) = log x + 1. Standard board MCQ.

2. [PYQ · March 2023] A function f is said to be increasing at a point c if _____.

(A) f'(c) = 0 **(B)** f'(c) > 0 (C) f'(c) < 0 (D) f'(c) = 1

Positive derivative at c means locally increasing. This is the definition the board expects.

3. [PYQ · March 2023] The integrating factor of dy/dx + y/x = x is _____.

(A) e^x (B) e^{-x} **(C)** x (D) -x

Linear DE: IF = e^∫(1/x)dx = e^(log x) = x.

4. [PYQ · March 2023] Corner points of the feasible region are (0,0), (3,0), (2,1), (0,7/3). Maximum value of z = 4x + 5y is _____.

(A) 12 **(B)** 13 (C) 35/3 (D) 0

z(3,0)=12, z(2,1)=13, z(0,7/3)=35/3≈11.7. Max 13 at (2,1).

5. [PYQ · March 2023] The sum due is also called _____.

(A) Face value (B) Present value (C) Cash value (D) True discount

Sum due = face value = amount written on the bill, due at maturity.

6. [PYQ · March 2023] byx is _____.

(A) Regression coefficient of y on x (B) Regression coefficient of x on y (C) Correlation coefficient between x and y (D) Covariance between x and y

The subscript reads 'y on x': slope of the regression line of y on x.

7. [PYQ · March 2023] The complicated but efficient method of measuring trend of a time series is _____.

(A) Graphical method (B) Method of moving averages **(C)** Method of least squares (D) Method of addition

Least squares fits the best-fit trend line; heavy computation, best precision.

8. [PYQ · March 2023] An agent places insurance of ₹4,00,000; premium is ₹35 per thousand per annum. His commission at 15% of premium is _____.

(A) ₹1,400 **(B)** ₹2,100 (C) ₹14,000 (D) ₹21,000

Premium = 400 × 35 = ₹14,000; commission = 15% of 14,000 = ₹2,100.

9. [PYQ · March 2023] Banker's discount is always _____ the true discount.

(A) less than (B) equal to **(C)** greater than (D) the reciprocal of

BD is interest on face value, TD on present worth; face value ≥ present worth, so BD ≥ TD.

10. [PYQ · March 2023] The number of days of grace legally allowed on a bill is _____.

(A) 2 **(B)** 3 (C) 5 (D) 7

Three days of grace are added to the nominal due date of a bill.

Flashcards — March 2023

Front	Recall line
Reduction method	Row-reduce [A|B], back-substitute from the bottom row
b_yx	Slope of the regression line of y on x; passes through ($\bar{x}$, $\bar{y}$)
Growth DE	dN/dt = kN; doubling time fixes k, then scale by doublings
LPP optimum	Always at a corner point of the feasible region
Area under $y^2 = 4x$	Double the top branch: $2\int 2\sqrt{x}\,dx$ over [0, a]
Max area, fixed perimeter	Equal sides — the square wins
Finding k (pmf)	$\Sigma P(x) = 1$; reject negative roots
BD vs TD	Banker's discount ≥ true discount, always