July 2025

Mathematics & Statistics (Commerce)

Paper J-388 · 80 marks · 3 hours — “Monsoon supplement: annuities, regression and one LPP”

8 key questions solved step by step · 10-question answer key · 8 flashcards

Official board question paper (board portal PDF), July 2025 supplementary - transcribed; solutions re-derived and machine-checked.

Section	Coverage
Section I	Q.1–Q.3 · Mathematical Logic, Matrices, Differentiation, Applications of Derivatives, Integration, Differential Equations
Section II	Q.4–Q.6 · Insurance & Annuity, Regression, Index Numbers, Time Series, LPP, Sequencing & Assignment, Probability Distributions

Key questions — solved in full

Q.2 (A) (ii) · 3 marks

Differentiation · Logarithmic derivative · medium

Q. Find dy/dx, if $y = (\log x)^x + x^5$.

1. $y = (\log x)^x + x^5 \Rightarrow dy/dx = d/dx(\log x)^x + d/dx(x^5)$

↳ Two separate functions sit here, so we differentiate each one on its own. The first part has x sitting in the exponent, so we use logarithmic differentiation. The second part is a plain power rule.

2. $u = (\log x)^x \Rightarrow \log u = x \log(\log x)$

↳ Take natural log of the first part only. Log of u equals x times log of log x. Now differentiate both sides with respect to x, keeping one upon u on the left.

3. $1/u du/dx = \log(\log x) + x \cdot (1)/(\log x) \cdot 1/x \Rightarrow du/dx = (\log x)^x [\log(\log x) + (1)/(\log x)]$

↳ Right side is a product, so the product rule gives log of log x plus one upon log x. Multiply both sides by u to land on du by dx.

4. $d/dx(x^5) = 5x^4$

↳ The second piece is friendly. Derivative of x to the fifth is five x to the fourth. Nothing more to do here, just park it aside for the final addition.

5. $dy/dx = (\log x)^x [\log(\log x) + (1)/(\log x)] + 5x^4$

↳ Join the two results and the derivative is complete. Examiners repeat this variable exponent question almost every year, so the take log first trick is worth full marks.

ANSWER

$dy/dx = (\log x)^x [\log(\log x) + (1)/(\log x)] + 5x^4$

Exam rule — Variable sitting in the exponent? Take logarithms first, then differentiate implicitly.

Q.2 (B) (i) · 4 marks

Applications of Derivatives · MPC, MPS, APC, APS · medium

Q. Find MPC, MPS, APC and APS, if the expenditure E_(c) of a person with income I is given as E_(c) = (0.0003) I² + (0.075) I, when I = 1000.

1. $MPC = (dE_(c))/(dI) = d/dI (0.0003 I^2 + 0.075 I) = 0.0006 I + 0.075$

↳ Expenditure here is consumption. Marginal propensity to consume means the derivative of expenditure with respect to income. Always differentiate the function first, substitution comes later.

2. $MPC = 0.0006(1000) + 0.075 = 0.675$

↳ Now put income equal to one thousand. Point zero zero zero six times one thousand is point six, plus the point zero seven five. MPC settles at point six seven five.

3. $MPS = 1 - MPC = 1 - 0.675 = 0.325$

↳ Income splits only into consumption and savings. So the marginal propensity to save is one minus MPC. One minus point six seven five gives point three two five.

4. $C = E_(c)(1000) = 0.0003(1000)^2 + 0.075(1000) = 300 + 75 = 375$

↳ For APC we need actual consumption at income one thousand. Substitute and collect. Three hundred from the square term plus seventy five from the linear term, so consumption is three hundred seventy five rupees.

5. $APC = 375/1000 = 0.375$, $APS = 1 - APC = 0.625$

↳ APC is consumption divided by income, three hundred seventy five upon one thousand, which is point three seven five. APS is one minus APC, so point six two five.

6. $MPC + MPS = 1$, $APC + APS = 1$ ✓

↳ Run the final check. MPC plus MPS is one, APC plus APS is one. If any pair misses one, retrace your arithmetic before moving to the next part.

ANSWER

MPC = 0.675, MPS = 0.325, APC = 0.375, APS = 0.625

Exam rule — MPC is dEc/dI at the given income; the saving part is always 1 minus the consumption part.

Q.2 (B) (ii) · 4 marks

Differential Equations · Homogeneous type · hard

Q. Solve the differential equation: $x^2 y\,dx - (x^3 + y^3)\,dy = 0$.

1. $x^2 y\,dx = (x^3 + y^3)\,dy \Rightarrow dx/dy = (x^3 + y^3)/(x^2 y) = x/y + (y^2)/(x^2)$

↳ Every term carries degree three, so this is a homogeneous equation. Here it is easier to treat x as a function of y. Rearrange and write dx by dy on the left.

2. $x = vy \Rightarrow dx/dy = v + y\,dv/dy$

↳ Standard substitution for homogeneous type. Put x equal to v times y, so dx by dy becomes v plus y times dv by dy. The v machinery does the work.

3. $v + y\,dv/dy = v + (1)/(v^2) \Rightarrow y\,dv/dy = (1)/(v^2)$

↳ Substitute into the right side. It becomes v plus one upon v squared. The plain v cancels from both sides and one clean line survives.

4. $\int v^2\,dv = \int dy/y \Rightarrow (v^3)/(3) = \log y + c$

↳ Separate the variables now. v squared dv goes left, dy upon y goes right. Integrate the left side by power rule and the right side as log y.

5. $v = x/y \Rightarrow (x^3)/(3y^3) = \log y + c$

↳ Bring back v as x upon y. The integral gives one upon three, x cubed upon y cubed, equals log y plus c. Substitute slowly here, this is where students slip.

6. $(x^3)/(y^3) - 3\log y = c$

↳ Multiply through by three and absorb the factor into the constant. The general solution reads x cubed upon y cubed minus three log y equals c, exactly what the board wants.

ANSWER

$(x^3)/(y^3) - 3\log y = c$ (general solution)

Exam rule — Homogeneous DE: every term same degree in x, y — substitute x = vy or y = vx, then separate.

Q.3 (A) (i) · 3 marks

Mathematical Logic · Converse, Inverse, Contrapositive · easy

Q. Write the converse, inverse and contrapositive of the following statement: 'If a man is bachelor, then he is happy.'

1. p: a man is bachelor, q: he is happy; statement $p \to q$

↳ First name the parts. p is a man is bachelor, q is he is happy. The given statement is p implies q, and we build all three relatives from these two.

2. Converse: $q \to p$ — 'If a man is happy, then he is a bachelor.'

↳ The converse simply swaps the two parts, q implies p. In words, if a man is happy, then he is a bachelor. No negation anywhere in this one.

3. Inverse: $\sim p \to \sim q$ — 'If a man is not a bachelor, then he is not happy.'

↳ The inverse negates both parts without swapping. Not p implies not q. If a man is not a bachelor, then he is not happy. Same order as the original.

4. Contrapositive: $\sim q \to \sim p$ — 'If a man is not happy, then he is not a bachelor.'

↳ The contrapositive swaps and negates together. Not q implies not p. If a man is not happy, then he is not a bachelor. This one never betrays the original truth value.

5. $p \to q \equiv \sim q \to \sim p$ (contrapositive)

↳ Remember the trio pattern for the exam. Converse swaps, inverse negates, contrapositive does both. And only the contrapositive is logically equivalent to the original statement.

ANSWER

Converse: If a man is happy, then he is a bachelor. Inverse: If a man is not a bachelor, then he is not happy. Contrapositive: If a man is not happy, then he is not a bachelor.

Exam rule — Converse swaps, inverse negates, contrapositive does both — and equals the original in truth.

Q.3 (A) (ii) · 3 marks

Matrices · Inverse by adjoint method · medium

Q. Find the inverse of bmatrix 3 & 1 & 5 2 & 7 & 8 1 & 2 & 5 bmatrix by adjoint method.

1. |A| = 3(35 - 16) - 1(10 - 8) + 5(4 - 7) = 57 - 2 - 15 = 40

↳ Adjoint method has three jobs, determinant first, then nine cofactors, then adjoint. Expand the determinant along row one and collect the three two by two minors.

2. C_{11} = +(35 - 16) = 19, C_{12} = -(10 - 8) = -2, C_{13} = +(4 - 7) = -3

↳ The determinant works out to fifty seven minus two minus fifteen, which is forty. Non zero, so the inverse exists. Now the row one cofactors, nineteen, minus two and minus three.

3. C_{21} = 5, C_{22} = 10, C_{23} = -5, C_{31} = -27, C_{32} = -14, C_{33} = 19

↳ Row two cofactors with the checkerboard signs, five, ten and minus five. Row three gives minus twenty seven, minus fourteen and nineteen. Nine numbers, done.

4. adj A = [C_(ji)] = bmatrix 19 & 5 & -27
-2 & 10 & -14
-3 & -5 & 19 bmatrix

↳ Adjoint is the transpose of the cofactor matrix. Read every row of cofactors as a column and the adjoint matrix stands ready, waiting only for the determinant forty.

5. A^{-1} = (1)/(|A|) adj A = 1/40bmatrix 19 & 5 & -27
-2 & 10 & -14
-3 & -5 & 19 bmatrix

↳ Inverse formula, A inverse equals adjoint upon determinant. Keep the one by forty outside the matrix rather than dividing each entry, it earns the method mark.

6. A (adj A) = |A| I = 40 I ✓

↳ Verification habit, multiply A with its adjoint. Diagonal entries become forty, off diagonal entries vanish. A adj A equals determinant times identity, so our adjoint is honest.

ANSWER

A^{-1} = 1/40bmatrix 19 & 5 & -27
-2 & 10 & -14
-3 & -5 & 19 bmatrix

Exam rule — Adjoint method: |A|, nine cofactors, transpose them; A^{-1} = adj A divided by |A|.

Q.5 (A) (i) · 3 marks

Insurance & Annuity · Sinking fund · medium

Q. A person wants to create a fund of ₹6,96,150 after 4 years at the time of his retirement. He decides to invest a fixed amount at the end of every year in a bank that offers him interest of 10% p.a. compounded annually. What amount should he invest every year? [Given $(1.1)^4$ = 1.4641]

1. A = (C [$(1+i)^n$ - 1])/(i), with A = 696150, i = 0.1, n = 4

↳ Fixed amount at the end of every year, that is an immediate annuity. The future value is known and we must find the yearly installment C.

2. $(1.1)^4$ = 1.4641 ⇒ $(1.1)^4$ - 1 = 0.4641

↳ The board has gifted the compound factor, one point one to the fourth equals one point four six four one. Subtract one to get point four six four one.

3. ($(1.1)^4$ - 1)/(0.1) = (0.4641)/(0.1) = 4.641

↳ Divide by the interest rate. Point four six four one upon point one is four point six four one. This is the annuity factor for four yearly deposits.

4. C × 4.641 = 696150

↳ Now the fund equation. C times four point six four one equals six lakh ninety six thousand one hundred fifty. One multiplication, one division, no drama.

5. C = (696150)/(4.641) = 150000 rupees

↳ Divide the target fund by four point six four one and it lands exactly on one lakh fifty thousand. He must invest one lakh fifty thousand rupees every year.

ANSWER

₹1,50,000 invested at the end of every year.

Exam rule — Immediate annuity: FV = C[(1+i)ⁿ − 1] / i; a given (1+i)ⁿ is a gift — use it.

Q.5 (A) (ii) · 3 marks

Regression · Line of Y on X · medium

Q. Following information is given about advertising expenditure (X) and sales (Y), both in lakh rupees: $\bar{x}$ = 10, $\bar{y}$ = 90, σ_x = 3, = 12 and the correlation coefficient between X and Y is 0.8. (a) Obtain the regression of Y on X. (b) What is the likely sales when the advertising budget is 15 lakh?

1. byx = r()/(σ_x) = 0.8 × 12/3 = 3.2

↳ Regression of Y on X needs the slope b y x. The formula is r times sigma y upon sigma x. Feed the values, point eight times twelve upon three, which is three point two.

2. Line of Y on X passes through ($\bar{x}$, $\bar{y}$) = (10, 90)

↳ Every regression line passes through the mean point. Here x bar is ten and y bar is ninety. Lock this point in your rough work, it anchors the line.

3. y - 90 = 3.2(x - 10) ⇒ y = 3.2x + 58

↳ Write the line in point slope form. y minus ninety equals three point two times, x minus ten. Simplify and the line reads y equals three point two x plus fifty eight.

4. y(15) = 3.2(15) + 58 = 48 + 58 = 106

↳ Estimation part now. The advertising budget is fifteen lakh, so substitute x equal to fifteen. Three point two times fifteen is forty eight, plus fifty eight gives one hundred six.

5. Likely sales = 106 lakh rupees

↳ So the likely sales are one hundred six lakh rupees. In commerce papers always carry the unit into the final statement, lakh rupees here, not just a bare number.

ANSWER

y = 3.2x + 58; when x = 15, likely sales = 106 lakh rupees.

Exam rule — b_yx = r·σy/σx; the y-on-x line passes through ($\bar{x}$, $\bar{y}$) — substitute only into it.

Q.6 (B) (ii) · 4 marks

Linear Programming · Graphical method · medium

Q. Maximize z = 60x + 50y subject to x + 2y ≤ 40, 3x + 2y ≤ 60, x ≥ 0, y ≥ 0.

1. x + 2y = 40: (40,0), (0,20); 3x + 2y = 60: (20,0), (0,30); shade below both

↳ Two resource lines and the first quadrant. Draw each boundary by its intercepts and shade below both lines, because both constraints are of the less than or equal type.

2. (3x + 2y) - (x + 2y) = 60 - 40 ⇒ 2x = 20 ⇒ (x, y) = (10, 15)

↳ Find where the two boundary lines cross. Subtract the first equation from the second, two x equals twenty, so x is ten and then y is fifteen.

3. Corners: (0,0),\ (20,0),\ (10,15),\ (0,20)

↳ The feasible region is the quadrilateral under both lines in the first quadrant. Four corners only, the origin, twenty zero, ten fifteen and zero twenty.

4. z(0,0) = 0, z(20,0) = 1200, z(10,15) = 1350, z(0,20) = 1000

↳ Corner tour with z equals sixty x plus fifty y. Origin gives zero. Twenty zero gives twelve hundred. Ten fifteen gives six hundred plus seven fifty, that is thirteen fifty. Zero twenty gives one thousand.

5. zmax = 1350 at (10, 15)

↳ The biggest value, thirteen hundred fifty, sits at ten fifteen. Corner point theorem again, the optimum of a linear programme always lives at a corner, never in the middle.

ANSWER

zmax = 1350 at x = 10, y = 15.

Exam rule — Graphical LPP: draw boundaries, list corner points, evaluate z at each — the optimum is at a corner.

Objective answer key — July 2025

1. [PYQ · July 2025] Matrix B = bmatrix 0 & 3 & 1 -3 & 0 & -4 P & 4 & 0 bmatrix is skew-symmetric. The value of P is _____.

(A) 1 **(B)** -1 (C) 0 (D) -3

Skew-symmetry needs B(3,1) = −B(1,3); here B(1,3) = 1, so P = −1. The other entries already agree.

2. [PYQ · July 2025] The area of the region bounded by the curve y = x², the X-axis and the lines x = 1 and x = 3 is _____.

(A) 3/26 sq. units (B) 3 sq. units (C) 26 sq. units **(D)** 26/3 sq. units

Area = $\int_1^3 x^2\,dx = [x^3/3]$ from 1 to 3 = (27 − 1)/3 = 26/3 square units.

3. [PYQ · July 2025] If the average revenue is 45 and elasticity of demand is 3, then the marginal revenue is _____.

(A) 15 **(B)** 30 (C) 45 (D) 60

MR = AR(1 − 1/η) = 45 × (1 − 1/3) = 45 × 2/3 = 30.

4. [PYQ · July 2025] The integrating factor (I.F.) of dy/dx + y = e^{-x} is _____.

(A) e^x (B) e^{-x} (C) x (D) 1

Linear DE with P = 1: I.F. = e^∫1 dx = e^x; then (e^x y)′ = e^x(y′ + y) = 1, so e^x works.

5. [PYQ · July 2025] The sum due is also called as _____.

(A) True discount **(B)** Face value (C) Present value (D) Cash value

The sum due is the amount written on the bill, payable at maturity — its face value.

6. [PYQ · July 2025] If P_{01}(L) = 120.4 and P_{01}(P) = 130.6, then P_{01}(D-B) is _____.

(A) 25.1 (B) 60.2 **(C)** 125.5 (D) 65.3

Dorbish-Bowley index is the arithmetic mean of Laspeyres and Paasche: (120.4 + 130.6)/2 = 125.5.

7. [PYQ · July 2025] F(x) is the c.d.f. of a discrete r.v. X taking values -2, -1, 0, 1, 2 with probabilities 0.2, 0.3, 0.15, 0.25, 0.1. Then F(-3) = _____.

(A) 1 (B) 0.2 (C) 0.15 **(D)** 0

F(−3) = P(X ≤ −3) = 0, because the smallest value X can take is −2.

8. [PYQ · July 2025] If X has Poisson distribution with parameter m and P(X = 3) = P(X = 4), then m = _____.

(A) 2 (B) 3 **(C)** 4 (D) 5

e^(−m)m^3/3! = e^(−m)m^4/4! cancels to 1/3! = m/4!, so m = 4!/3! = 4.

9. [Practice · Binomial Distribution] A pair of dice is thrown 3 times. If getting a doublet is considered a success, the probability of exactly two successes is _____.

(A) ⅙ **(B)** 5/72 (C) 25/216 (D) 1/36

p = 6/36 = 1/6; P(X = 2) = $^3C_2(1/6)^2(5/6)$ = 15/216 = 5/72, as in the paper's Q.6 (A) (iii).

10. [Practice · Regression Coefficients] The equations of two regression lines are 8x - 10y + 66 = 0 and 40x - 18y = 214. Then ($\bar{x}$, $\bar{y}$) = _____.

(A) (13, 17) (B) (17, 13) (C) (12, 15) (D) (13, 15)

Both regression lines pass through the mean; solving 8$\bar{x}$ − 10$\bar{y}$ = −66 with 40$\bar{x}$ − 18$\bar{y}$ = 214 gives (13, 17).

Flashcards — July 2025

Front	Recall line
MPC + MPS	Always 1; MPC = dEc/dI at the given income
Log-differentiation	y = (log x)^x: take log first, then differentiate
Adjoint method	A^{-1} = adj A / |A|; adj A is the cofactor transpose
Homogeneous DE	Same degree in x and y; put x = vy, separate, integrate
Sinking fund	FV = C[(1+i)^n − 1] / i, deposits at year end

$b_{yx} \times b_{xy}$	$= r^2$; choose the sign of the root from r itself
LPP optimum	Occurs at a corner of the feasible region, never the centre
Contrapositive	Swap and negate; logically equivalent to the original