July 2024

Mathematics & Statistics (Commerce)

Paper J-178 · 80 marks · 3 hours — “Supplementary sitting, same syllabus, sharper numbers”

8 key questions solved step by step · 10-question answer key · 8 flashcards

Official board question paper (board portal PDF), July 2024 supplementary - transcribed; solutions re-derived and machine-checked.

Section	Coverage
Section I	Q.1–Q.3 · Logic, Matrices, Differentiation, Integration, Applications of Derivatives, Differential Equations
Section II	Q.4–Q.6 · Commission, Insurance, Annuities, Index Numbers, LPP, Assignment, Regression, Time Series, Probability

Key questions — solved in full

Q.2 (A) (ii) · 3 marks

Matrices · Condition $(A+B)^2 = A^2 + B^2$ · hard

Q. If A = bmatrix 1 & 2 -1 & -2 bmatrix, B = bmatrix 2 & a -1 & b bmatrix and if $(A+B)^2 = A^2 + B^2$, find the values of a and b.

1. $(A+B)^2 = A^2 + AB + BA + B^2 = A^2 + B^2 \Rightarrow AB + BA = O$

↳ Watch this beauty. One small condition, A plus B whole squared equals A squared plus B squared. That happens only when the cross terms vanish, so let us expand and compare entry by entry.

2. A+B = bmatrix 3 & a+2
-2 & b-2 bmatrix, $(A+B)^2$ = bmatrix 5-2a & (a+2)(b+1)
-2-2b & b^2-4b-2a bmatrix

↳ First add the matrices. A plus B has entries three, a plus two in the top row and minus two, b minus two below. Then square it, row into column, and keep the four products tidy.

3. A^2 = bmatrix -1 & -2
1 & 2 bmatrix, B^2 = bmatrix 4-a & 2a+ab
-2-b & -a+b^2 bmatrix

↳ Now the other side. Square A alone, then square B alone, and add them. A squared comes to minus one minus two on top, one and two below. B squared keeps the letters a and b.

4. 5-2a = 3-a ⇒ a = 2; -2-2b = -1-b ⇒ b = -1

↳ Set the two results equal, entry against entry. Top left gives five minus two a equal three minus a, so a is two. Bottom left gives minus two minus two b equal minus one minus b, so b is minus one.

5. (a+2)(b+1) = 4 × 0 = 0 ✓, b^2-4b-2a = 1 ✓

↳ Cross check the remaining entries before you box it. Top right and bottom right both settle perfectly with these values. The pair a equal two, b equal minus one satisfies every entry, so the answer stands verified.

ANSWER

a = 2, b = -1

Exam rule — $(A+B)^2 = A^2 + B^2$ forces AB = BA; here expand entry-wise and compare both sides.

Q.2 (B) (ii) · 4 marks

Definite Integration · By parts · medium

Q. Evaluate : $\int_1^3 \log x \, dx$

1. $\int_1^3 \log x \cdot 1 \, dx$, first function = log x (ILATE)

↳ Log x has no direct integral formula, so we use integration by parts. Take log x as the first function and one as the second. Remember the order, log always goes first.

2. $\int \log x \, dx = x\log x - \int x \cdot 1/x \, dx = x\log x - x$

↳ By parts gives first times integral of second, minus integral of derivative of first times integral of second. That is x log x minus the integral of one. Clean.

3. $[x\log x - x]_1^3 = (3\log 3 - 3) - (0 - 1)$

↳ Put the limits on x log x minus x. At three, three log three minus three. At one, one times log one, that is zero, minus one gives plus one. Watch the sign flip.

4. $= 3\log 3 - 2 \approx 1.296$

↳ Three log three minus three plus one. Three log three minus two, about one point three. Log of one always dies at the lower limit, use that shortcut freely in the exam.

ANSWER

$3\log 3 - 2 \approx 1.296$

Exam rule — $\int \log x \, dx = x \log x - x + c$; at the lower limit 1 the value is −1, so it adds back.

Q.3 (A) (ii) · 3 marks

Indefinite Integration · Partial fractions · medium

Q. Evaluate : $\int (2x+1)/(x(x-1)(x-4)) \, dx$

1. $(2x+1)/(x(x-1)(x-4)) = A/x + (B)/(x-1) + (C)/(x-4)$

↳ Denominator already factored, three simple factors. Perfect case for partial fractions. Split the fraction as A by x, B by x minus one, C by x minus four.

2. A = (1)/((0-1)(0-4)) = ¼, B = (3)/(1(1-4)) = -1, C = (9)/(4(4-1)) = ¾

↳ Cover up trick saves time. Put x equal zero, A is one quarter. Put x equal one, B is three over minus three, minus one. Put x equal four, C is nine over twelve, three quarters.

3. ∫ = ¼∫ dx/x - ∫ (dx)/(x-1) + ¾∫ (dx)/(x-4)

↳ Now integrate each simple piece. Integral of A by x is A log x, and so on for the other two. Three logs, three constants.

4. = ¼log x - log(x-1) + ¾log(x-4) + c

↳ One quarter log x, minus log of x minus one, plus three quarters log of x minus four, plus c. Always differentiate back mentally to check the original numerator returns.

ANSWER

¼log x - log(x-1) + ¾log(x-4) + c

Exam rule — Cover-up method: A = fraction value at each root with its own factor hidden. Fastest board-safe route.

Q.3 (A) (iii) · 3 marks

Definite Integration · Area under parabola · easy

Q. Find the area of the region bounded by $y^2 = 25x$ and the line $x = 4$.

1. $y^2 = 25x \Rightarrow y = \pm 5\sqrt{x}$; at $x = 4$, $y = \pm 10$

↳ The parabola y squared equals twenty five x opens rightward. The line x equals four cuts it at y plus and minus ten. The region sits symmetric about the x axis.

2. $A = 2 \int_0^4 5\sqrt{x}\, dx = 10\int_0^4 x^{(1/2)}\, dx$

↳ Total area is twice the upper branch. Twice the integral of five root x from zero to four. Strip width is dx along the x axis.

3. $A = 10\ [\frac{2}{3}x^{(3/2)}]_0^4 = 20/3\ (8)$

↳ Root x integrates to two thirds x power three by two. Ten times two thirds, twenty over three. At four, four to the three by two is eight.

4. A = 160/3 ≈ 53.33\ sq units

↳ Twenty over three times eight, one hundred sixty by three, about fifty three point three three square units. Same shape as the March paper, different numbers, identical method.

ANSWER

A = 160/3 sq units ≈ 53.33

Exam rule — Parabola $y^2 = kx$ capped at x = a: area = $2 \int \sqrt{(kx)}\, dx$ from 0 to a.

Q.5 (A) (i) · 3 marks

Annuities · Immediate annuity rate · medium

Q. Find the rate of interest compounded annually if an immediate annuity of ₹20,000 per year amounts to ₹41,000 in 2 years.

1. $A = P\ ((1+i)^n - 1)/(i)$, P = 20000, n = 2, A = 41000

↳ Immediate annuity means each yearly payment earns interest till the end. Two payments of twenty thousand rupees each. First grows for one year, second grows not at all. Total forty one thousand.

2. $((1+i)^2 - 1)/(i) = 41000/20000 = 2.05$

↳ Substitute. Twenty thousand times one plus i squared minus one over i equals forty one thousand. Cancel twenty thousand, the bracket over i must be two point zero five.

3. $(1+i)^2 - 1 = 2i + i^2 \Rightarrow (i(2+i))/(i) = 2 + i = 2.05$

↳ Expand the numerator, one plus two i plus i squared minus one. That is i times two plus i. The i cancels beautifully, leaving two plus i equal two point zero five.

4. i = 0.05 ⇒ rate = 5% p.a.; 20000(1.05) + 20000 = 41000 ✓

↳ So i equals zero point zero five, and that is a rate of five percent per annum. Sanity check, five percent on twenty thousand is one thousand rupees, total forty one thousand.

ANSWER

Rate of interest = 5% per annum (i = 0.05).

Exam rule — Annuity amount A = P[(1+i)ⁿ − 1]/i; for small n, expand (1+i)ⁿ and the i cancels.

Q.5 (B) (iii) · 4 marks

Linear Programming · Graphical method · medium

Q. Solve the following L.P.P. by graphical method: Maximize z = 7x + 11y, subject to 3x + 4y ≤ 24, 5x + 3y ≤ 30, x ≥ 0, y ≥ 0.

1. 3x+4y = 24 (axes 8, 6), 5x+3y = 30 (axes 6, 10); shade the ≤ side

↳ Draw both boundary lines as equalities. Three x plus four y equal twenty four meets the axes at eight and six. Five x plus three y equal thirty meets them at six and ten. Shade below both.

2. Corners: O(0,0), A(6,0), B(0,6), P = (3x+4y=24) ∩ (5x+3y=30)

↳ The feasible quadrilateral has four corners. Origin, six on the x axis, six on the y axis, and the crossing point of the two lines. Find that crossing by elimination.

3. 15x+20y=120 and 15x+9y=90 ⇒ 11y = 30 ⇒ y = 30/11, x = 48/11

↳ Multiply the first equation by five, the second by three, subtract. Eleven y equal thirty, so y is thirty by eleven. Back substitute, x lands on forty eight by eleven.

4. z(O)=0, z(6,0)=42, z (48/11, 30/11) = 666/11 ≈ 60.5

↳ Now the corner tour with z equal seven x plus eleven y. Origin gives zero. Six zero gives forty two. The crossing point gives six hundred sixty six over eleven, about sixty point five.

5. z(0,6) = 66 ⇒ zmax = 66 at (0, 6)

↳ Last corner, zero six. z equals eleven times six, sixty six. That beats everyone, so the maximum sits on the y axis. Corner point theorem wins again.

ANSWER

zmax = 66 at x = 0, y = 6.

Exam rule — Corner Point Theorem: list every corner, evaluate z, pick the winner — the y-axis corner won here.

Q.6 (A) (i) · 3 marks

Regression · Coefficients and estimation · easy

Q. For a bivariate data, $\bar{x}$ = 53, $\bar{y}$ = 28, byx = -1.2, b_{xy} = -0.3. Find : (a) correlation coefficient between x and y, (b) estimate y for x = 50.

1. r^2 = byx · b_xy = (-1.2)(-0.3) = 0.36

↳ Two regression coefficients are given, so the correlation sits in between them. Multiply them first. Minus one point two times minus zero point three gives plus zero point three six.

2. r = ± 0.6; both b's negative ⇒ r = -0.6

↳ Square root of zero point three six is zero point six. Sign choice now. Both regression coefficients are negative, so r must also be negative. Minus zero point six, a fair negative correlation.

3. y - $\bar{y}$ = byx(x - $\bar{x}$) ⇒ y - 28 = -1.2(x - 53)

↳ For estimation use the regression of y on x, the one with slope b y x. Line through the mean point fifty three comma twenty eight.

4. y = 91.6 - 1.2x; y(50) = 91.6 - 60 = 31.6

↳ Simplify to y equal ninety one point six minus one point two x. Put x equal fifty. Minus one point two times minus three adds three point six, so y is thirty one point six.

ANSWER

r = -0.6; estimate y(50) = 31.6.

Exam rule — r = ±√(b_yx · b_xy), sign follows the b's; estimate y only from the y-on-x line.

Q.6 (B) (ii) · 4 marks

Binomial Distribution · Defectives sampling · medium

Q. There are 10% defective items in a large bulk of items. What is the probability that a sample of 4 items will include not more than one defective item?

1. n = 4, p = 1/10, q = 9/10

↳ Large bulk means the chance stays fixed, so binomial applies. Success is a defective item, probability one by ten. Sample has four items, so n is four and q, the good item chance, is nine by ten.

2. $P(X \le 1) = P(0) + P(1) = q^4 + C(4, 1)\, p\, q^3$

↳ Not more than one defective means zero defectives or exactly one. Write both binomial probabilities separately, q to the fourth power, and four p q cubed.

3. $P(0) = (0.9)^4 = 0.6561$, $P(1) = 4(0.1)(0.9)^3 = 0.2916$

↳ Nine by ten to the fourth gives zero point six five six one. Then four times one by ten times nine by ten cubed gives zero point two nine one six.

4. $P(X \le 1) = 0.6561 + 0.2916 = 0.9477$

↳ Add them, zero point nine four seven seven. Almost ninety five percent chance the sample is clean or nearly clean. Board loves this phrasing, not more than one, so remember the two cases.

ANSWER

P(X ≤ 1) = 0.9477

Exam rule — 'Not more than one' = P(0) + P(1); large bulk keeps p constant, so binomial is valid.

Objective answer key — July 2024

1. [PYQ · July 2024] Which of the following sentences is a statement in logic?

(A) He is a good actor. (B) Did you eat lunch yet? **(C)** Every real number is a complex number. (D) Bring the motor car here.

A statement must be definitely true or false. The others are opinion, question and command, so only the mathematical sentence qualifies.

2. [PYQ · July 2024] If $y = 2x^2 + \log 2 + 5$, then dy/dx = ?

(A) x **(B)** 4x (C) 2x + log 2 (D) -4x

Derivative of $2x^2$ is 4x; log 2 and 5 are constants, so their derivative is zero.

3. [PYQ · July 2024] If $x = 2at^2$ and $y = 4at$, then dy/dx = ?

(A) $-(1)/(2at^2)$ (B) $(1)/(2at^3)$ **(C)** 1/t (D) $(1)/(4at^3)$

$dy/dt = 4a$ and $dx/dt = 4at$, so $dy/dx = 4a$ over $4at$, which is 1/t.

4. [PYQ · July 2024] $\int_{-2}^{3} (dx)/(x+5)$ = ?

(A) -log (8/3) (B) 3log (⅜) **(C)** log (8/3) (D) -2log (⅜)

The integral is log of x plus five, evaluated from minus two to three: log eight minus log three, that is log of eight by three.

5. [PYQ · July 2024] The order and degree of the differential equation $(d^2x)/(dt^2) + (dx/dt)^2 + 8 = 0$ are ____.

(A) order = 2, degree = 2 (B) order = 1, degree = 2 (C) order = 1, degree = 1 **(D)** order = 2, degree = 1

The highest order derivative is second, so order is 2. Its power is one, so degree is 1; the square sits on the first derivative only.

6. [PYQ · July 2024] The equation of the tangent to the curve $y = x^2 + 4x + 1$ at P(-1, -2) is ____.

(A) 2x - y = 0 (B) x + 2y + 5 = 0 (C) 2x + 4 = 3y (D) 5x + y = 1

Slope $y' = 2x + 4$ gives 2 at $x = -1$; tangent $y + 2 = 2(x + 1)$ simplifies to $2x - y = 0$.

7. [PYQ · July 2024] Paasche's Price Index Number is given by ____.

(A) $(\sum p_0 q_0)/(\sum p_1 q_0) \times 100$ (B) $(\sum p_0 q_1)/(\sum p_1 q_1) \times 100$ (C) $(\sum p_1 q_0)/(\sum p_0 q_0) \times 100$ **(D)** $(\sum p_1 q_1)/(\sum p_0 q_1) \times 100$

Paasche's index uses current year quantities q_1 as weights: current year prices over base year prices on q_1, times 100.

8. [PYQ · July 2024] If E(X) = 4 and X follows Poisson's distribution, then V(X) = ?

(A) 2 (B) -2 **(C)** 4 (D) -4

For a Poisson distribution the mean and variance are both equal to m. Since E(X) = 4, the variance is also 4.

9. [PYQ · July 2024] If an agent charges 12% commission on the sales of ₹48,000, then his total commission is ₹____.

(A) ₹4,800 **(B)** ₹5,760 (C) ₹5,600 (D) ₹4,080

Commission is 12 percent of 48,000, that is 0.12 × 48,000 = 5,760 rupees.

10. [Practice · Regression Coefficients] If byx = -1.2 and $b_x y$ = -0.3, then the correlation coefficient r = ?

(A) 0.6 **(B)** -0.6 (C) 0.36 (D) -0.36

r = ±√(b_yx · b_xy) = ±√0.36 = ±0.6; both regression coefficients are negative, so r = −0.6.

Flashcards — July 2024

Front	Recall line
$(A+B)^2 = A^2 + B^2$	Only when AB = BA; expand and equate entries
$\int \log x\, dx$	x log x − x + c; at x = 1 it equals −1
Partial fractions	(2x+1)/x(x−1)(x−4) = A/x + B/(x−1) + C/(x−4)
Area under $y^2 = 25x$	$2\int 5\sqrt{x}\, dx$ from 0 to 4 = 160/3 sq units
LPP optimum	Corner Point Theorem — test z at every corner
r from b_yx and b_xy	r = ±√(b_yx · b_xy); sign follows the b's
Immediate annuity	$A = P[(1+i)^n - 1]/i$; expand when n = 2
Binomial, at most 1	$P(X \le 1) = q^n + npq^{n-1}$; 0.9477 for n=4, p=0.1